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Reaching the Ultimate Efficiency of Solar Energy Harvesting with a Nonreciprocal Multijunction Solar Cell

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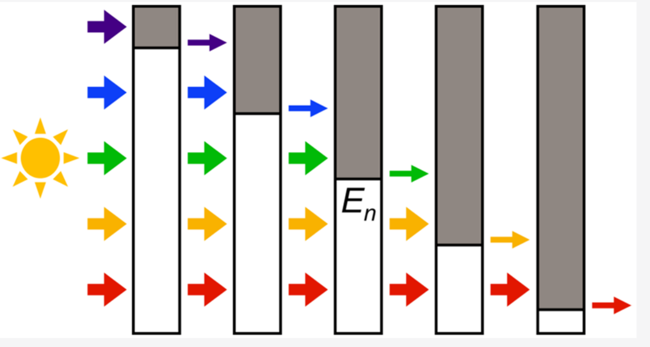
ABSTRACT: The Landsberg limit represents the ultimate efficiency limit of solar energy harvesting. Reaching this limit requires the use of nonreciprocal elements. The existing device configurations for attaining the Landsberg limit, however, are very complicated. Here, we introduce the concept of a nonreciprocal multijunction solar cell and show that such a cell can reach the Landsberg limit in the idealized situation where an infinite number of layers are used. We also show that such a nonreciprocal multijunction cell outperforms a standard reciprocal multijunction cell for a finite number of layers. Our work significantly simplifies the device configuration required to reach the ultimate limit of solar energy conversion and points to a pathway toward using nonreciprocity to improve solar energy harvesting.



KEYWORDS: *multijunction solar cell, efficiency, nonreciprocity, Landsberg limit*

Understanding the fundamental limits of solar energy conversion and developing device configurations to reach these limits have been of central importance in the study of solar cells.[1−3] When reciprocity is assumed, the efficiency of solar energy conversion has an upper bound called the multicolor limit.[2] With the use of nonreciprocal components, one can further increase the efficiency beyond the multicolor limit to reach the Landsberg limit,[4] which is defined by an efficiency

$$\eta_L = 1 - \frac{4}{3}\frac{T_C}{T_S} + \frac{1}{3}\left(\frac{T_C}{T_S}\right)^4 \tag{1}$$

Here, T_S and T_C are the temperatures of the sun and the solar cell, respectively; $\eta_L = 93.3\%$ assuming $T_S = 6000$ K and $T_C = 300$ K. The Landsberg limit represents the ultimate efficiency limit of solar energy conversion, as can be proven using fundamental thermodynamic considerations.[2,5]

The multicolor limit can be reached with use of multijunction solar cells, which consists of a stack of multilayers of semiconductors with different band gaps. Compared with the multijunction solar cell, however, the existing proposed configurations for reaching the Landsberg limit are far more complicated. The first of such proposals, by Ries, consists of an array of circulators and Carnot engines (Figure 1a).[5] Green showed that the same efficiency can be reached by replacing each Carnot engine in Ries' configuration with a reciprocal multijunction cell with infinite number of layers (Figure 1b).[2,3] Subsequently, using optical devices that exhibit nonreciprocity[6,7] in the reflection processes, Green proposed an alternative configuration without the need of circulators (Figure 1c).[8] This configuration uses an array of elements, each consisting of a reflective nonreciprocal device and a reciprocal multijunction cell with infinite number of layers.

In recent years, there has been further developments in the design of nonreciprocal optical devices. In contrast to the developments of common nonreciprocal devices such as circulators and isolators, which focus on achieving contrast between various reflection or transmission processes, it was noted that complete contrast can be achieved between the absorption and thermal emission processes through the breaking of Kirchhoff's law.[9−19] In particular, it has been pointed out in ref 19 that Kirchhoff's law[20] can be broken in semitransparent structures, so that under ideal operation an absorber can fully absorb light from one direction while directing the emitted light solely to the opposite side.

In this paper, motivated by the results in ref 19, we consider the configuration of a nonreciprocal multijunction solar cell, as shown in Figure 3. In this configuration, similar to the standard multijunction solar cells, the layers are semiconductors with

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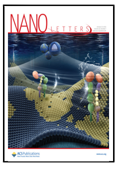

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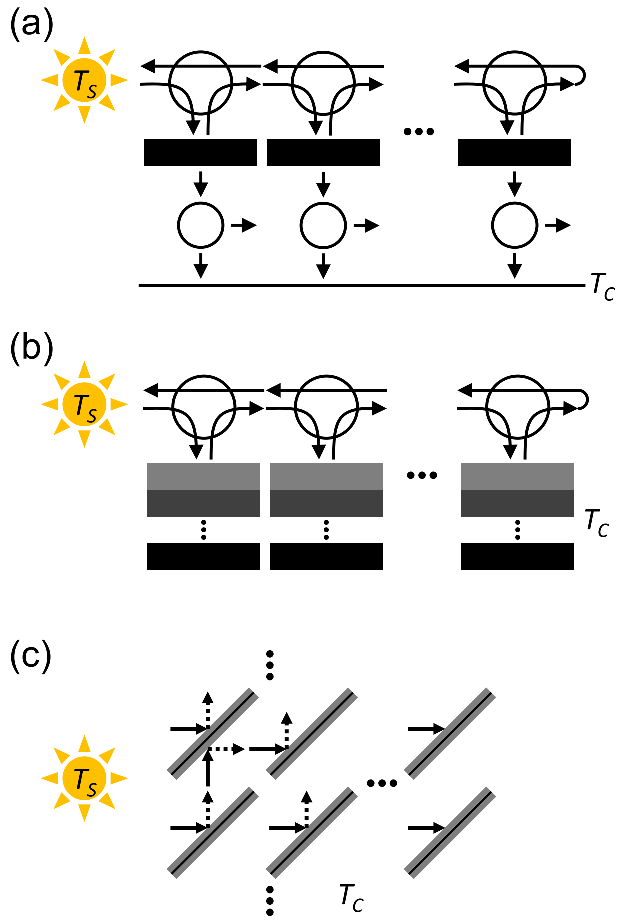

Figure 1. Various previously proposed device configurations for reaching the Landsberg limit. (a) Design of Ries based on optical circulators and Carnot engines.[5] (b) Design of Green based on optical circulators and tandem cells with infinite number of layers.[2] (c) Design of Green based on nonreciprocal elements functioning in reflection-based setup.[8]

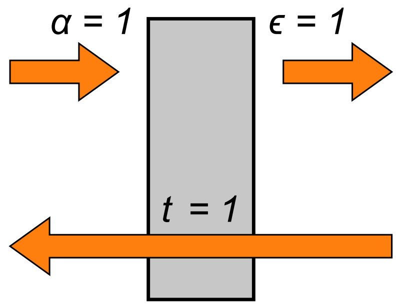

Figure 2. Illustration of the semitransparent nonreciprocal absorber. α, ϵ, and t each refers to absorptivity, emissivity, and transmissivity, respectively. Under ideal operation, the absorber absorbs light perfectly from one side, emits solely to the other, and transmits completely in the opposite direction.

We consider a nonreciprocal multijunction cell (Figure 3), where each layer has the property of being a semitransparent nonreciprocal absorber as described above, and we set up the theoretical formalism for treating this cell. In Figure 3, each layer is a semiconductor with a band gap E_i for i = 1, 2,..., M. We assume that $E_{i+1} > E_i$. The sunlight is directly incident on layer M. The setup here is similar to the standard multijunction cell, with, however, one critical difference: each layer completely absorbs all above-band-gap photons from the left and emits only to the right. Each absorbed photon produces an electron−hole pair. Photons with energy below the band gap are not absorbed and instead transmit to the next layer. We consider the quasi-Fermi levels to be constant throughout each semiconductor layer.[2] The difference between the electron quasi-Fermi level and the hole quasi-Fermi level is equal to qV, where q is the elementary charge and V is the operating voltage of the layer. Then the chemical potential of the photons emitted by the layer is qV according to refs 21−23. Under these assumptions, when recombinations taking place are completely radiative and the radiation pattern of the light emission from the layers follow that of a blackbody, a flux of photons emitted by a layer at temperature T with energy between E_l and E_m can be expressed as[22,24]

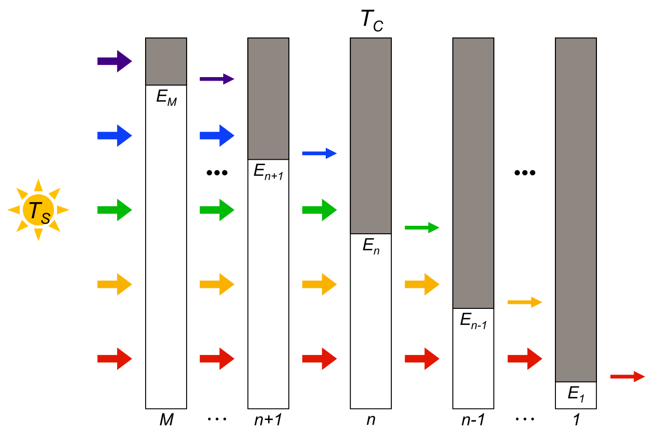

Figure 3. Illustration of our nonreciprocal multijunction solar cell. Each bar represents a semiconductor layer in a cell, where the height of the white region indicates the band gap of the corresponding layer. Sunlight is incident from the left, and each layer is transparent for the light with energy below the band gap, as expressed by colored thick arrows. Light emitted from each layer heads only to the right side, as shown with thin arrows.

different band gaps, with the larger band gap semiconductor placed closer to the front side of the cell facing the sun. Unlike the standard multijunction solar cells, however, here, each layer has the nonreciprocal semitransparent absorption/emission properties as discussed above. We show that this configuration, in the ideal limit with infinite number of semiconductor layers, can reach the Landsberg limit. Compared with previous device configurations proposed for reaching the Landsberg limit, our configuration here is far more compact. We also show that the nonreciprocal multijunction cells outperform its reciprocal counterparts for any number of layers greater than one. The performance difference grows as the number of layers increases, and a significant performance difference occurs when the number of layers exceeds 5.

We start by a brief review of ref 19. The work introduced a semitransparent nonreciprocal absorber (Figure 2). In this absorber, light incident from the left side is perfectly absorbed by the structure, whereas light incident from the right side is perfectly transmitted through the structure. By the second law of thermodynamics, one can then show that the structure has zero emissivity to the left side, and unity emissivity to the right side. Reference 19 showed that such an absorber can be constructed using gyrotropic materials.

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$$N(E_l, E_m, V, T) = \frac{2\pi}{h^3c^2}\int_{E_l}^{E_m}\frac{E^2}{\exp\left(\frac{E-qV}{kT}\right)-1}dE \tag{2}$$

Here, h, c, and k are the Planck constant, the speed of light in vacuum, and the Boltzmann constant, respectively.

Current collected from the n-th layer is determined by the difference between the generation and recombination rates[25] and, thus, can be expressed by using the function N of eq 2 as following for layers $n = 1, 2,..., M - 1$:

$$I_n = qA[f_s N(E_n, E_{n+1}, 0, T_S) + f_c N(E_{n+1}, \infty, V_{n+1}, T_C) - f_c N(E_n, \infty, V_n, T_C)] \tag{3}$$

and for layer M

$$I_M = qA[f_s N(E_M, \infty, 0, T_S) - f_c N(E_M, \infty, V_M, T_C)] \tag{4}$$

Here, A is the area of the layer and V_i for $i = 1, 2,..., M$ is the operating voltage of the i-th layer. We set the temperature of the sun as T_S and that of the cell as T_C. f_s and f_c are geometrical parameters that indicate the solid angle corresponding to the fraction of a hemisphere, over which illumination from the sun (f_s) or emission from the semiconductors (f_c) is occurring.[26] Both of these parameters are set to unity in further analysis as, in this paper, we are considering theoretical limit, which is reached in the fully concentrated case. In eq 3, the first term shows the absorption of photon flux coming from the sun. We note that the chemical potential of these photons is zero. Photons with energy above the band gap E_n are absorbed by the n-th semiconductor layer, but within the solar spectrum, only photons with energy below E_{n+1}, which is the band gap of the previous layer, can arrive at this layer. The second term represents the absorption of photon flux that is emitted from the $(n + 1)$-th layer; hence the chemical potential is V_{n+1} and the temperature is T_C. The last term expresses the emission of photon flux from the n-th layer to the $(n - 1)$-th layer. The interpretation of the first and second terms inside the bracket of eq 4 is similar to that with the first and the last terms in eq 3, respectively.

The last term in eq 3 can be written as

$$N(E_n, \infty, V_n, T_C) = N(E_n, E_{n+1}, V_n, T_C) + N(E_{n+1}, \infty, V_n, T_C) \tag{5}$$

from which eq 3 can be rewritten as

$$\frac{I_n}{A} = \frac{2\pi q}{h^3c^2}\int_{E_n}^{E_{n+1}}\left[\frac{E^2}{\exp\left(\frac{E}{kT_S}\right)-1} - \frac{E^2}{\exp\left(\frac{E-qV_n}{kT_C}\right)-1}\right]dE + \frac{2\pi q}{h^3c^2}\int_{E_{n+1}}^{\infty}\left[\frac{E^2}{\exp\left(\frac{E-qV_{n+1}}{kT_C}\right)-1} - \frac{E^2}{\exp\left(\frac{E-qV_n}{kT_C}\right)-1}\right]dE \tag{6}$$

Using the formalism above, we first derive the theoretical limit for the efficiency of the case in which there are an infinite number of layers in the cell; that is, M is infinite. For a layer with a certain band gap E, we assume that its adjacent previous layer has a band gap of $E + dE$, where dE is infinitesimal. Suppose the operating voltage of this layer is $V(E)$. Under these assumptions, power generated from this layer can be expressed as follows:

$$\frac{dP}{A} = \frac{2\pi q}{h^3c^2}\left[\frac{E^2V(E)}{\exp\left(\frac{E}{kT_S}\right)-1} - \frac{E^2V(E)}{\exp\left(\frac{E-qV(E)}{kT_C}\right)-1}\right]dE + \frac{2\pi q}{h^3c^2}V'(E)V(E)dE\int_E^{\infty}\frac{\frac{q}{kT_C}\exp\left(\frac{E'-qV(E)}{kT_C}\right)E'^2}{\left[\exp\left(\frac{E'-qV(E)}{kT_C}\right)-1\right]^2}dE' \tag{7}$$

where the current is determined using eq 6 assuming that $E_{n+1} - E_n$ is small. By substituting E and V with $x \cdot kT_C$ and $y \cdot \frac{kT_C}{q}$, respectively, eq 7 can be simplified as following:

$$\frac{dP}{A} = \frac{2\pi(kT_C)^4}{h^3c^2}\cdot\left[\frac{x^2y}{\exp\left(\frac{T_C}{T_S}x\right)-1} - \frac{x^2y}{\exp(x-y)-1} + \dot{y}y\int_x^{\infty}\frac{\exp(x'-y)x'^2}{[\exp(x'-y)-1]^2}dx'\right]dx \tag{8}$$

where $\dot{y} = \frac{dy}{dx}$. If we set the terms in bracket as $f(y, \dot{y}, x)$, the problem of maximizing the power becomes a problem of finding a "path" $y(x)$ between $x = x_1$ and $x = x_2$ that maximizes $\int_{x_1}^{x_2} f(y, \dot{y}, x)$. The optimal $y(x)$ can then be determined by solving

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0 \tag{9}$$

as can be derived using the same variational techniques commonly found in Lagrangian mechanics.[27] Solving eq 9 leads to the following result:

$$y(x) = \left(1 - \frac{T_C}{T_S}\right)x \rightarrow V(E) = \frac{E}{q}\left(1 - \frac{T_C}{T_S}\right) \tag{10}$$

Subsituting eq 10 back into eq 8 and dividing the result by the total power input from the sun P_S leads to the efficiency. When we use the following equation for P_S

$$\frac{P_S}{A} = \frac{2\pi}{h^3c^2}\int_0^{\infty}\frac{E^3}{\exp\left(\frac{E}{kT_S}\right)-1}dE \tag{11}$$

the efficiency limit is calculated as

$$\eta = 1 - \frac{4}{3}\frac{T_C}{T_S} + \frac{1}{3}\left(\frac{T_C}{T_S}\right)^4 \tag{12}$$

which is exactly the Landsberg limit of eq 1. Since the Landsberg limit represents the ultimate limit of solar energy harvesting, we have shown that an ideal nonreciprocal multijunction cell can operate as an ultimate solar energy harvester.

We now provide a more detailed comparison between the reciprocal and nonreciprocal multijunction cells for varying number of semiconductor layers. For the reciprocal case, we consider a configuration shown in Figure 4, as discussed in refs 26 and 28. Similar to the system shown in Figure 3, here, the reciprocal system also consists of multiple semiconductor layers at different band gaps. Since the layers are reciprocal, each layer emits to both sides. References 26 and 28 propose to add band-pass filters between the semiconductor layers. The

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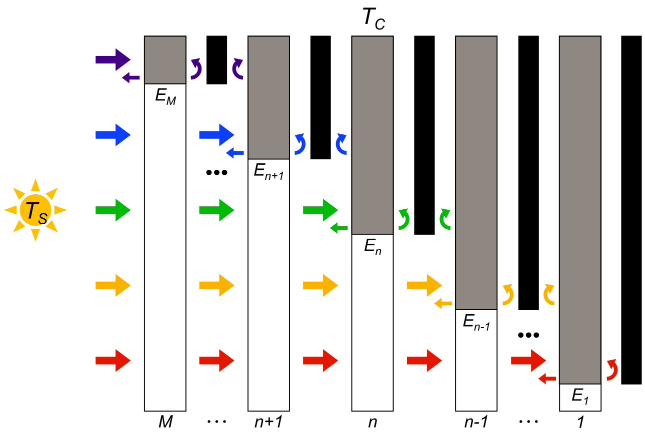

Figure 4. Illustration of a reciprocal multijunction solar cell with low energy pass filters. Light emitted from each layer, as shown with the thin arrows, can head to both sides, but photons with high energy are reflected by the filters and reabsorbed by the layer.

filter between the n-th and $(n + 1)$-th layer reflects the photons with energy above the band gap energy E_{n+1} of the $(n + 1)$-th layer and transmits the photons with energy below E_{n+1}. For reciprocal systems, when the number of layers is finite, such a filter is important for improving the system efficiency, as it ensures that the photons with energy above E_{n+1} to be harvested in the $(n + 1)$-th layer where the band gap of the semiconductor is higher. For the nonreciprocal systems shown in Figure 3, each layer is transparent for photons incident from the right as we discuss in Figure 2. Therefore, placing such a filter has no effect on the efficiency.

We compute the efficiency of the reciprocal multijunction cell in the same way as in refs 26 and 28. For the nonreciprocal multijunction cell, power from each layer is the current of eq 6 times the operating voltage of each layer, and total power is simply the sum of all of these. The band gaps and operating voltages of the layers are optimized so that the total generated power is maximized.

Figure 5 compares the efficiencies of the reciprocal and the nonreciprocal cases as the number of layers varies. Here, we assume T_S = 6000 K and T_C = 300 K. We note that with a single layer, the limiting efficiency is equal to the Shockley–Queisser limit[1] of 40.8% for both cases. With two layers, the nonreciprocal case (56.1%) exceeds the reciprocal case (55.8%) by 0.3 percentage points. With five layers, the efficiency improvement is over 1 percentage point. Ultimately, the difference between the two cases can reach up to 6.5 percentage points as the reciprocal case approaches the multicolor limit of 86.8% and the nonreciprocal case approaches the Landsberg limit of 93.3% with increasing number of layers. Therefore, with few layers, nonreciprocity may not influence the efficiency enhancement greatly, but the impact grows as the number of layers increases.

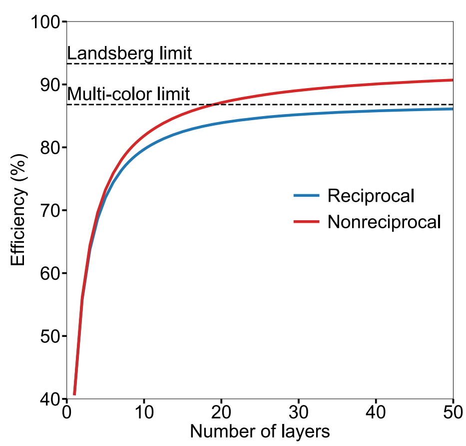

Figure 5. Plot of efficiency limit versus the number of layers in a cell. Blue and red curves express the efficiency of reciprocal and nonreciprocal multijunction cells, respectively. Multicolor limit and Landsberg limit are marked in the figure.

In this work, we focus on finding out the theoretical limit that can be achieved for fully concentrated light under the assumption that the nonreciprocal absorbers can function ideally. The assumptions include the absorbers being able to absorb and emit light in a nonreciprocal manner for all 2π steradian solid angles and for unpolarized light. We note that if absorbers like the one designed in ref 19 are to be used as the layers here, the incidence angle of the light has to be kept normal to the cell, which is not the case for full concentration. However, even for the case in which the light is not concentrated, the Landsberg limit can be reached if one restricts the angular range of absorption of the cell such that $f_s = f_c$. Moreover, the result in ref 19 is limited to transverse magnetic (TM)-polarized case. For unpolarized incident light, to employ the structures shown in ref 19, one can use two sets of nonreciprocal cells, together with a polarization splitter that splits the two polarizations to the two sets of the cells and a polarization rotator in front of one of the cells to ensure that the TM polarizations are incident on both cells. Alternatively, it would be of interest to design nonreciprocal absorber structures that operate for both polarizations.

To realize the potential of such a nonreciprocal multijunction cell requires the development of a semiconductor-based nonreciprocal semitransparent absorber. For instance, one can imagine a design that incorporates magneto-optical materials into semiconductor solar cells to achieve nonreciprocal semitransparent absorbers. An example of the design may be close to that of ref 19 but with the lossless reciprocal dielectric regions replaced by an absorbing semiconductor material. In this design, key challenges are reducing the parasitic loss associated with the magneto-optical material and determining the proper semiconductor material.

To summarize, we propose the concept of a nonreciprocal multijunction solar cell. In the limit with infinite number of layers, such a cell can reach the Landsberg limit, which is the ultimate limit of solar energy harvesting. The configuration of our cell is far simpler compared with that of previous proposed device configurations aiming to reach the Landsberg limit. Our work thus represents a step forward in the understanding of the fundamental physics of solar energy conversion. We also show that such a nonreciprocal multijunction cell outperforms standard reciprocal multijunction cells for a finite number of layers. Hence, our work points to a pathway toward using nonreciprocity to improve solar energy harvesting in practice.

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Notes

The authors declare no competing financial interest.

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